Bending Lab

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1. Introduction

2. Methods

Firstly, in order to simulate the simple beam setup, 2 chairs of approximately the same height were set up next to each other. A pencil and a pen were placed on each chair and then a Rolson® 30cm stainless steel ruler was placed on top. The pen and pencil were placed in order to allow the ruler to bend freely. Then, a piece of string was tied around the neck of a 100ml measuring cylinder and hung from the ruler to act as the load and cause the bending in the ruler.

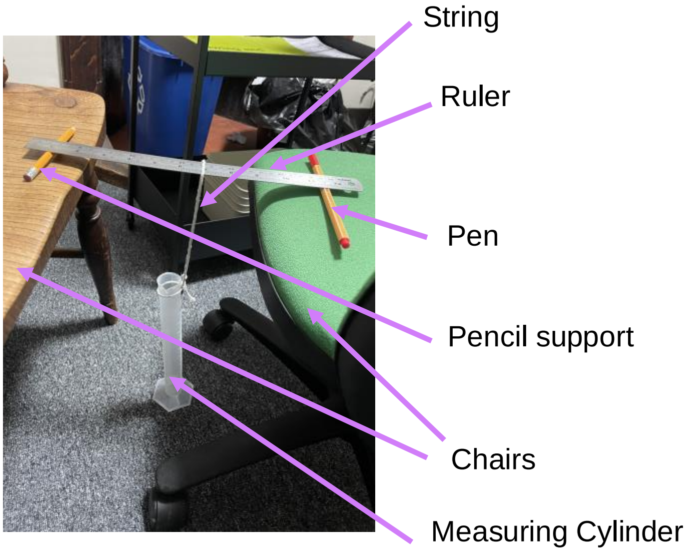

Figure 1 - Experimental setup

The positions on the ruler at which it rested on the pen and pencil were recorded and used to determine the midpoint and the span length of the ruler. The string holding the measuring cylinder was then moved to rest at the midpoint.

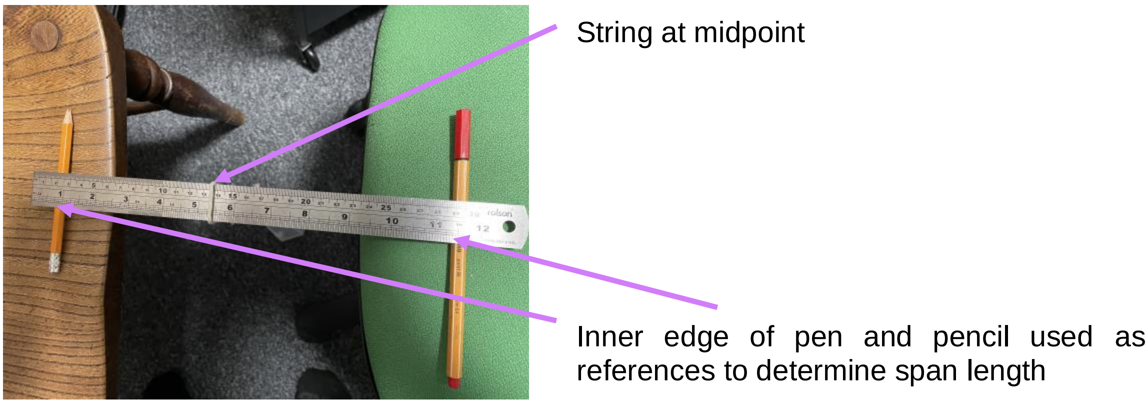

Figure 2 - Top view of set up showing span length

An initial set of deflection readings were taken with 0mL of water in the measuring cylinder by measuring, with the tape measure, the distance from the floor to a defined point on the ruler. This was then repeated for the other side of the ruler at the same point. By averaging these two readings, the influence of any torque applied to the experimental setup was eliminated.

Figure 3 - Image showing how to position tape measure to measure distance to ground

Then, a cup was filled with tap water and a 60mL plastic syringe was used to take 30mL of water and inject it into the measuring cylinder to increase the load on the ruler. Measurements from the point on the edge of the ruler to the ground were repeated for both sides. This process was repeated again for 60, 90 and 120mL of water in the measuring cylinder. The measuring cylinder was then emptied and the whole experiment was repeated another two times in order to determine a mean and to reduce the influence of random errors. Finally, the breadth and depth of the ruler were measured using Vernier Callipers.

3. Results

3.1. Finding *M*

Equation 1 was used to find the constant M:

$$\delta = M\frac{PL^3}{EI} \tag{1}$$

Where M is a constant, P is the applied load, L is the span length of the ruler (between the pen and the pencil), I is the second moment of area of the ruler's cross section, E is the Young's modulus of the ruler and δ is the deflection of the ruler.

Equation 1 can be compared to the equation for a straight line:

$$y=ax+b \tag{2}$$

Where a is the gradient of the plot of variable y against variable x, and b is the intercept of the line on the y-axis. By plotting the deflection of the ruler (δ) against the product of the applied load and the span length cubed, divided by the product of the Young's modulus and the second moment of area ($\frac{PL^3}{EI}$), the gradient of the line can be extracted, which should be equal to the constant $M.$

Equation 3 helps determine the applied load (P):

$$P = mg \tag{3}$$

Where m is the mass of water in the cylinder and g is the gravitational acceleration at the surface of the Earth which is defined as 9.81 ms^{-2}.[1]
1mL of water is equivalent to 1g of water so this can be used to determine m.

Table 1- Distances between the ruler and the ground for the corresponding mass of water in the cylinder for each ruler for each trial and the averages obtained

Mass of Water (g)	0	30	60	90	120
Trial 1					
Distance 1 (cm)	44.0	43.9	43.8	43.7	43.5
Distance 2 (cm)	44.1	44.0	43.8	43.8	43.6
Average 1 (cm)	44.05	43.95	43.80	43.75	43.55
Trial 2					
Distance 1 (cm)	44.0	44.0	43.9	43.9	43.8
Distance 2 (cm)	44.0	44.0	43.8	43.8	43.7
Average 2 (cm)	44.00	44.00	43.85	43.85	43.75
Trial 3					
Distance 1 (cm)	44.1	44.0	43.9	43.7	43.7
Distance 2 (cm)	44.2	44.1	43.9	43.8	43.7
Average 3 (cm)	44.15	44.05	43.90	43.75	43.70
Average (cm)	44.07	44.00	43.85	43.78	43.67

The average distance between the ruler and the ground in the last row of the table can be used to find the deflection (δ) for each mass of water by subtracting each distance from the distance when the mass is 0.

Table 2 – Deflection (δ) for the corresponding mass of water

Mass of Water (g)	0	30	60	90	120
Deflection (cm)	0.00	0.07	0.22	0.29	0.4

Equation 4 can be used to find the second moment of area of the ruler's cross section:

$$I = \frac{bh^3}{12} \tag{4}$$

Where b is the breadth of the steel ruler found to be 2.54cm and h is the depth of the ruler found to be 0.8mm.

The Young's modulus of stainless steel (E) is 195 GPa[2] and the span length of the beam (L) was found to be 27cm.

Now all data is available to plot the graph.

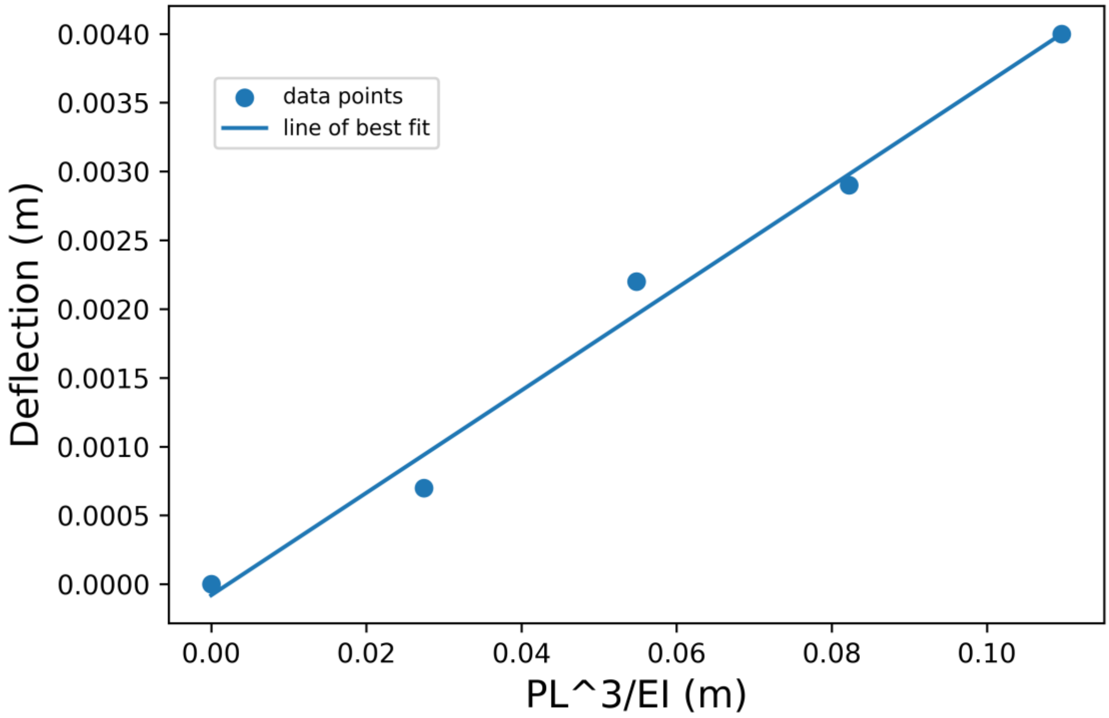

Figure 4 - Graph produced

The gradient of the line of best fit is 0.0358 so the constant M is 0.0358.

3.2. Design Problem

Rearranging equation 1 and putting it into the context of the steel beam of the hospital roof:

$$I = \frac{MPL^3}{E\delta} \tag{5}$$

Where I is the second moment of area of the beam's cross section, M is a constant, 0.0358, P is the applied load, 50kN, L is the length of beam between two supports, 25m, E is the Young's modulus of the beam, 200GPa, and δ is the deflection of the beam, which can be no more than 140mm.

Providing the beam with a maximum deflection of 140mm, equation 5 gives I as 998.88x10^{-6} m^4.

Rearranging equation 4 and putting it into the context of the steel beam of the hospital roof:

$$h = \sqrt[3]{\frac{12I}{b}} \tag{6}$$

Where h is the depth of the beam, I is the second moment of area of the beam's cross section, 998.88x10^{-6} m^4, and b is the breadth of the beam, 200mm.

Equation 6 gives h as 391mm so the required depth of the beam which keeps the deflection to no more than 140mm is 391mm.

4. Discussion

Through experimentation, the constant M was found to be 0.0358. This constant was then used to calculate the required breadth a steel beam would need to have to distribute the load from a corrugated roof sheeting to the walls of a hospital as part of the emergency repairs this hospital as going through after a destructive earthquake. This breadth was found to be 391mm so the beam would need to have at least this breadth in order to stay within the tolerances provided.

When looking at what dimensions steel beams are available to purchase, there is one with a depth of 364mm and a breadth of 173 and another with a depth of 406 and breadth 178.[3] These are very close to the result obtained, hence it is fully feasible to have a steel beam with a depth of 391mm and a width of 200mm.

There are many potential sources of error in this experimental process. Firstly, when measuring the distance between the ground and the ruler with the tape measure, the point where the tape measure touches the ground was always changing. This could lead to inaccuracies due to the tape measure not always measuring the exact vertical height. This error could be reduced by marking a point on the floor and always placing one end of the tape measure at this point. Another point of error is the string holding the measuring cylinder slipping off of the midpoint of the ruler which would lead to changes in the amount of load the ruler was being subject to. This error could have been reduced by securing the cylinder to the ruler with something more permanent or drilling a hole in the ruler at the midpoint and threading the string through that hole.

References

1. *Fundamental Physical Constants* (no date) *Codata value: Standard acceleration of gravity*. Available at: https://physics.nist.gov/cgi-bin/cuu/Value?gn
2. *AISI Type 304 Stainless Steel* (no date) *AISI Type 304 stainless steel*. Available at: https://www.matweb.com/search/datasheet_print.aspx?matguid=bd20a4281ae3430d97cfbebf6904ec50
3. *Universal structural steel beams* (2023) *Universal Beams*. Available at: https://www.steelbeamsuppliers.co.uk/products/universal-beams/